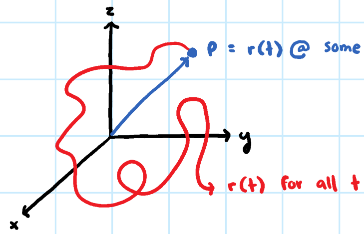
Integrals & Trajectories

Monday, May 22, 2023 8:55 AM

recall: $\vec{r}(t)$ = trajectory
$\vec{v}(t) = \vec{r}(t)'$ = velocity
$\vec{a}(t) = \vec{r}(t)''$ = acceleration

vectors w/ functions of t



$r(t) = \langle \cos(t), \sin(t), e^t \rangle$

- for each value of t, we get vector

question:

where particle initially

1) what if we know $\vec{r}(0)$ and velocity ($\vec{v}(t)$) @ all times? $\longrightarrow$ find $r(t)$ (when integrate once → need 1 constant → $\vec{r}(0)$)

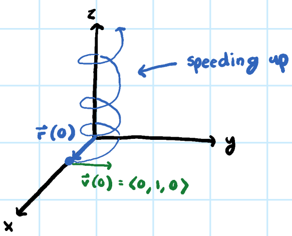
2) if we know $\vec{r}(0)$ and $\vec{v}(0)$, can we find $r(t)$ if $a(t)$ known? (when integrate twice → need 2 constants → $\vec{r}(0)$ & $\vec{v}(0)$)

ex 1) particle starts @ $\vec{r}(0) = \langle 1, 0, 0 \rangle$ & has velocity $\vec{v}(t) = \langle -\sin(t), \cos(t), t \rangle$. find $r(4\pi)$ = position @ time $t = 4\pi$.

solution: since $\vec{v}(t) = \vec{r}(t)'$, we have $\vec{r}(t) = \underbrace{\int \vec{v}(t)}_{\text{indefinite integral}} + c$

3 equations, 1 per component

vector!!! $\langle c_1, c_2, c_3 \rangle$



$$\vec{r}(t) = \int \langle -\sin(t), \cos(t), t \rangle \, dt$$

integral $\int$ goes in

$$= \langle -\int \sin(t), \int \cos(t), \int t \rangle + \langle c_1, c_2, c_3 \rangle$$

$$= \langle \cos(t), \sin(t), \frac{t^2}{2} \rangle + \langle c_1, c_2, c_3 \rangle$$

different constants

$$\vec{r}(t) = \langle \cos(t) + c_1, \sin(t) + c_2, \frac{t^2}{2} + c_3 \rangle$$

to find c_1, c_2, c_3 we use $\vec{r}(0) = \langle 1, 0, 0 \rangle$

it gives:

$$\vec{r}(0) = \langle \cos 0 + c_1, \sin 0 + c_2, \frac{0^2}{2} + c_3 \rangle = \langle 1, 0, 0 \rangle$$

thus $c_1 = 0$, $c_2 = 0$, $c_3 = 0$ ← in general some #'s, not always 0

$$\vec{r}(t) = \langle \cos(t), \sin(t), \frac{t^2}{2} \rangle$$

@ $t = 4\pi$:

$$\vec{r}(4\pi) = \langle \cos(4\pi), \sin(4\pi), \frac{(4\pi)^2}{2} \rangle = \langle 1, 0, \frac{16\pi^2}{2} \rangle = \langle 1, 0, 8\pi^2 \rangle$$